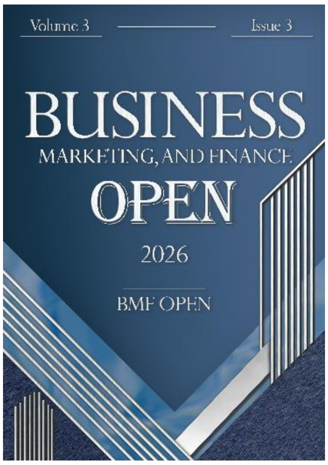
Empirical and Comparative Analysis of the Efficiency of Nonlinear Option Pricing Models (Quantum-Based and Heston) and Conventional Linear Models (Black–Scholes and Binomial Tree) in the Tehran Stock Exchange



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Abstract: Accurate option pricing, as one of the key tools for risk management and price discovery, requires the use of models capable of adapting to nonlinear behaviors and structured market volatilities. The present study aims to empirically evaluate the efficiency of both linear and nonlinear option pricing models in the Iranian market by examining four analytical frameworks: Black–Scholes and the Binomial Tree as linear models, and Heston and an extended quantum model based on the nonlinear Schrödinger equation as nonlinear models. The research data include 30 actively traded call option contracts listed on the Tehran Stock Exchange, each with a minimum of 70 trading days during the period from 2016 to 2022. In the numerical solution section, the quantum and Heston models were implemented using the method of lines and the fourth-order Runge–Kutta algorithm in a Python programming environment. The comparative evaluation criteria consisted of Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). Out-of-sample results indicate that, under most conditions—particularly for at-the-money (ATM) options and in high-volatility environments—the quantum model provides the most accurate estimates. The Heston model demonstrates competitive performance at medium maturities and moderate volatility levels; the Binomial Tree model performs comparably to the benchmark over short horizons; and the Black–Scholes model, overall, exhibits the lowest accuracy. Moreover, employing nonlinear frameworks not only reduces pricing deviation but also leads to relative savings in computational costs compared to linear iterations based on implied volatility extraction. This study is also distinguished from a computational perspective, as it represents the first application in Iranian option pricing literature that integrates the method of lines and Runge–Kutta within a Python-based numerical implementation, moving beyond the traditional reliance on regression-based and curve-fitting techniques. Although the limited depth of Iran's derivatives market, trading halts, and the absence of a comprehensive structured data repository remain research challenges, the findings suggest that employing nonlinear models—particularly those based on quantum dynamics—can provide a novel pathway toward fair pricing, the design of effective hedging strategies, and the advancement of financial engineering in the Iranian market.

Keywords: Option Pricing, Black–Scholes Model, Binomial Tree Model, Heston Model, Quantum Model (Nonlinear Schrödinger), Method of Lines, Runge–Kutta, Python.

1. Introduction

Option markets have evolved from niche instruments for hedgers into central venues for aggregating beliefs, transferring risk, and discovering forward-looking information about volatility and tail risk, especially in emerging exchanges where structural breaks and liquidity cycles are common [1]. Classical linear frameworks such as Black–Scholes and lattice methods remain the canonical baseline for pedagogy and benchmarking, yet their assumptions—constant volatility, lognormal returns, and continuous trading without frictions—are frequently violated in markets characterized by episodic illiquidity, policy shocks, and volatility clustering [2]. In the Tehran Stock Exchange (TSE), these departures from idealized dynamics are amplified by regime shifts, market microstructure constraints, and behavioral feedbacks, raising a practical question: which pricing paradigm—linear or nonlinear—delivers more reliable and operationally efficient valuations for listed options across varied market conditions [3]?

From a theoretical vantage, linear models provide closed-form intuition but often require ex post adjustments—smiles, skews, and term-structure overlays—to align with observed premia, whereas nonlinear and stochastic-volatility models embed such features in their generative dynamics [2]. The Black–Scholes framework yields tractability and analytic greeks; however, its constant-volatility postulate is inconsistent with stylized facts such as volatility clustering, leverage effects, and heavy tails that appear persistently in Iranian equity series [4]. Discrete-time lattice methods, particularly the Cox–Ross–Rubinstein binomial tree, approximate the continuous dynamics and allow for path-dependent features, yet their accuracy hinges on time-step selection and parameter fitting to implied surfaces that themselves are unstable across regimes [3]. The TSE literature echoes these concerns, documenting material gaps between linear-model valuations and traded option prices under changing volatility regimes and liquidity stresses [5].

Stochastic-volatility models, notably Heston, were proposed to bridge this gap by endogenizing variance as a mean-reverting diffusion correlated with price innovations, thereby producing smiles and skews directly from the structural specification [2]. In the Iranian context, stochastic-differential formulations have been used to model index dynamics and option values, demonstrating empirical advantages over constant-volatility assumptions in both in-sample fit and out-of-sample stability [6]. Extensions such as double-Heston with jumps further improve calibration to extreme-move risk, fitting both short-maturity skew and long-horizon term structures observed in local data [7]. Complementary numerical work comparing Monte Carlo and finite-difference discretizations for barrier-style products highlights the trade-offs between bias, variance, and computational cost when implementing stochastic-volatility engines in practice [8]. Earlier finite-difference schemes adapted to stochastic volatility within Iran's engineering literature also underlined stability conditions and boundary treatments essential for robust valuation under market frictions [4].

Beyond stochastic volatility, a growing strand of research draws analogies between financial diffusion and quantum dynamics, positing that market prices—subject to feedback, crowding, and phase-like regime changes—may be better represented by nonlinear wave-like equations than by purely diffusive linear ones [9]. Foundational studies have mapped the structural proximity of the Black–Scholes partial differential equation to the Schrödinger equation under change of variables, thereby motivating quantum-inspired generalizations that incorporate effective potentials and nonlinearities capturing interaction effects [10]. Subsequent work formulated European option pricing via a nonlinear Schrödinger (NLS) approach, introducing cubic terms to emulate self-interaction and adaptive market response, and reported improved calibration to observed surfaces in stressed regimes [11].

Building on this line, recent contributions proposed a nonlinear Black–Scholes model explicitly grounded in quantum dynamics, with empirical demonstrations of enhanced fit and better reproduction of smile dynamics relative to linear baselines [12]. Continued refinement has emphasized quantum-dynamics engines for nonlinear pricing, focusing on solvability, stability, and data-driven parameter identification in practical implementations [13].

Methodologically, quantum-inspired finance intersects with advances in quantum neural computation and hybrid architectures, enabling richer function approximation for pricing and risk under nonstationarity [14]. Hybrid quantum–neural models have been proposed for financial prediction tasks, demonstrating that quantum state representations and entanglement-inspired operators can improve learning capacity on nonconvex loss landscapes typical of financial time series [15]. On the numerical side, explicit algorithmic pipelines using fourth-order Runge–Kutta have been deployed to solve nonlinear Schrödinger-type formulations for stock pricing applications, balancing accuracy with implementational simplicity while retaining stability for moderate step sizes [16]. These developments are directly relevant to the empirical evaluation of nonlinear pricing engines on TSE options, where reliable PDE/ODE solvers and calibration routines must perform under data limitations and episodic regime shifts.

Iran-specific evidence provides a mixed but instructive picture regarding model choice. Comparative studies on TSE call options have shown that while Black–Scholes and binomial trees provide reasonable baselines, their performance deteriorates for at-the-money and short-dated contracts during high-volatility phases, with implied-volatility inversions indicating mis-specification [3]. Research on the profitability and relative behavior of European, American, and Asian options in Tehran suggests that payoff structure interacts with market frictions and volatility regimes, affecting the relative bias of linear models across moneyness and maturities [5]. Meanwhile, work on the general index using Heston SDEs demonstrates that stochastic-volatility features, including mean reversion and price–volatility correlation, are empirically salient in Iran's market, supporting the migration toward nonlinear state dynamics for pricing and hedging [6]. In applied practice, graduate-level theses and technical reports have implemented Heston-based pricing for follower (taba'i) securities, documenting feasibility and outlining calibration heuristics under local data constraints [17].

The broader asset-pricing context also matters. A deeper theoretical understanding of equilibrium-based models like the Capital Asset Pricing Model (CAPM) indicates that systematic risk premia are sensitive to information frictions and state-dependent beliefs, which, in turn, alter the shape of risk-neutral distributions that underlie option prices [1]. When cross-sectional crash-risk dependence is modeled via conditional copula–GARCH frameworks, empirical linkages emerge between tail co-movements and rational pricing structures, reinforcing the need for pricing engines that internalize higher-order dependence and volatility-of-volatility effects—features better accommodated by stochastic and nonlinear PDEs than by constant-volatility forms [18]. Cross-industry international comparisons likewise report heterogeneous performance of pricing models across technology-linked equities versus traditional sectors, underscoring that model adequacy is conditional on the underlying return process and volatility regime [19].

From a numerical-analysis perspective, the accuracy–cost frontier is central. Applied stochastic analysis provides guidelines for discretization, stability, and error control when transitioning from PDEs to implementable ODE systems or Monte Carlo estimators, particularly under stiff or nonlinear dynamics [2]. In practice, lattice models scale as O(N^2) in time–space nodes for vanilla claims, while Fourier, finite-difference, and method-of-lines solvers for Heston and NLS-type equations require careful boundary conditioning and time-stepping to mitigate numerical dispersion and ensure convergence of the Greeks [4]. Empirical studies in Iranian applications comparing Monte Carlo and finite-difference schemes for double-barrier structures illustrate that discretization choices induce systematic biases that can exceed model-choice effects when step control is inadequate [8]. This underscores a pragmatic point: any fair comparison of linear versus nonlinear pricing must harmonize solver accuracy and calibration protocols to avoid attributing numerical artifacts to model structure.

Conceptually, the theoretical bridge between Schrödinger-type dynamics and Black–Scholes highlights how potential functions and nonlinear terms can encode market microstructure phenomena—order-flow imbalance, feedback trading, and liquidity spirals—within the pricing operator itself [9]. Early demonstrations of quantum methods in option valuation confirmed that wave-like dynamics can reproduce surface features that otherwise require ad hoc parameterizations in linear settings [10]. The nonlinear Schrödinger approach incorporates a cubic interaction capturing self-reinforcing behaviors, which may correspond to transient herding or momentum ignition, and several studies report improvements in fitting volatility smiles and skew persistence using such terms [11]. More recent conference and preprint contributions have formalized the quantum-dynamics foundation of nonlinear Black–Scholes, offering calibration schemas and numerical recipes suitable for production environments [12, 13]. Parallel advances in quantum neural computation and hybrid architectures provide avenues for learning effective potentials directly from data, blending mechanistic PDEs with data-driven surrogates to improve robustness under nonstationarity [14, 15].

Within the Iranian market microstructure, regime-dependent volatility, temporary price limits, and episodic liquidity droughts demand models that are both adaptive and structurally expressive. Empirical work on Iranian derivatives reiterates that implied-volatility surfaces are non-flat and time-varying, often exhibiting asymmetries related to leverage and crash-risk premia [5]. Under such conditions, stochastic-volatility models like Heston have an intrinsic advantage because they generate smiles endogenously via variance dynamics and price–variance correlation, offering more coherent hedging ratios across strikes and maturities than ad hoc smile-adjusted Black–Scholes implementations [6]. Yet, when volatility-of-volatility and feedback behaviors become pronounced—such as during stress episodes—nonlinear quantum-inspired PDEs may further improve pricing accuracy by allowing state-dependent amplification or damping in the propagation of pricing information through the potential term and cubic nonlinearity [11, 13].

The choice of numerical engine is equally consequential for operational deployment. Fourth-order Runge–Kutta integrators applied to the method of lines provide a tractable route for solving both Heston-transformed systems and NLS-type equations with competitive accuracy–cost profiles, provided that spatial grids and boundary conditions are tuned to option payoff geometry [16]. This approach integrates well with calibration loops that minimize squared pricing errors (or robust alternatives) against observed premiums, aligning with best practices in applied stochastic analysis for ill-posed inverse problems [2]. For lattice benchmarks, step-adaptive binomial trees remain valuable as pedagogical references and for sanity checks, but their discrete nature can imprint stair-step artifacts on price surfaces, complicating direct comparison to smooth market quotes during high-volatility windows [3]. Iranian studies on complex payoffs further caution that solver selection interacts with barrier geometry and monitoring frequency, reinforcing the need for consistent numerical treatment across models in comparative research [8].

Importantly, empirical assessments should not occur in a theoretical vacuum: macro–micro linkages such as conditional crash-risk dependence and rational-pricing structures alter the state prices embedded in options, and ignoring these can lead to model misspecification masquerading as numerical error [18]. Cross-market evidence suggests that sectoral composition—e.g., technology-heavy cohorts—exhibits distinct volatility–jump profiles, implying that model performance is context-dependent and time-varying [19]. As such, comparative studies that span boom, correction, and high-volatility subperiods, and that report both statistical fit (RMSE, MAE, R^2) and formal forecast-comparison tests, are better positioned to make credible inferences about the superiority of one class over another [2, 3].

Against this backdrop, the present article situates TSE-listed European call options within a unified comparative framework that evaluates four representative models—Black–Scholes, binomial tree, Heston, and a nonlinear quantum (Schrödinger-type) specification—under consistent data, calibration, and numerical protocols. The investigation is anchored in prior Iranian applications of stochastic-volatility and numerical schemes [3, 4, 6-8, 17], and leverages recent advances in quantum-inspired pricing and computation [9-16], while grounding solver choices in the principles of applied stochastic analysis [2]. In doing so, it also acknowledges the broader asset-pricing context that shapes state-price densities in emerging markets [1] and the empirical realities of crash-risk dependence in Iranian equities [18], as well as comparative international evidence on model performance heterogeneity across sectors [19].

To empirically compare the predictive accuracy, stability, and computational efficiency of Black–Scholes, binomial tree, Heston, and nonlinear quantum (Schrödinger-type) option pricing models for European call options listed on the Tehran Stock Exchange across distinct market regimes using harmonized calibration and numerical methodologies

2. Methodology

The present study aims to conduct an empirical and comparative analysis of the efficiency of four option pricing models, including two conventional linear models (Black–Scholes and Binomial Tree) and two advanced nonlinear models (Heston and Quantum), in the Tehran Stock Exchange. The research framework was designed based on a comparison of predictive accuracy, stability, and computational efficiency among these models to determine which one best aligns with the real behavior of the Iranian market. The selection of these four models was made to capture the theoretical evolution from the assumption of constant volatility to stochastic dynamics and, subsequently, nonlinear and adaptive market behaviors—thus providing a comprehensive picture of the developmental trajectory of option pricing models.

In this study, the Black–Scholes model was first employed as the theoretical starting point of option pricing. This model, based on the assumption of constant volatility and the geometric Brownian motion of the underlying asset price, derives a closed-form solution for the value of a European call option through solving its partial differential equation (Black & Scholes, 1973). Alongside it, the Cox–Ross–Rubinstein binomial tree model uses a discrete approach by simulating possible price paths in separate time intervals to compute the option price. These two models, due to their computational simplicity, are of great importance in teaching and theoretical analysis; however, their constant volatility assumption causes their predictive accuracy to decline in real-world emerging markets like Iran, particularly when confronted with phenomena such as volatility smiles and structural shifts in volatility.

Subsequently, the Heston model, as one of the stochastic volatility frameworks, was used to address the shortcomings of classical models. In this model, variance is treated as a stochastic variable with mean-reverting properties, thereby representing the dynamic nature of volatility over time (Heston, 1993). By incorporating the correlation between price shocks and volatility shocks, this model can better explain empirical market phenomena, although computational complexity and parameter sensitivity in unstable market conditions may limit its efficiency. To further capture nonlinear behaviors, the quantum option pricing model—based on the Schrödinger equation and the probabilistic interpretation of quantum mechanics—was employed (Brody, 2006; Rublowski, 2022). In this model, the market is conceived as a dynamic environment with behavioral potential, and the underlying asset price is modeled as a particle oscillating within this potential field under behavioral and structural forces. This framework enables modeling of price jumps, behavioral phases, and nonlinear market dynamics and is expected to be more compatible with the characteristics of the Iranian capital market than classical and stochastic models.

To provide the mathematical foundations of the applied models and clarify the structural differences between linear and nonlinear approaches, the main equations of the four option pricing models—serving as the basis for numerical analysis and empirical comparison—are presented below.

The classical Black–Scholes model is one of the most widely used analytical frameworks for valuing European call options. In this model, it is assumed that the underlying asset price follows a geometric Brownian motion process, and variables such as volatility, interest rate, and dividend yield remain constant over time. The Black–Scholes formula for call option price is defined as follows:

Equation 1: $C = S \cdot N(d_1) - e^{-r(T-t)} \cdot N(d_2)$

Equation 2: $d_1 = [\ln(S/K) + (r + \sigma^2/2)(T-t)] / [\sigma\sqrt{(T-t)}]$

Equation 3: $d_2 = d_1 - \sigma\sqrt{(T-t)}$

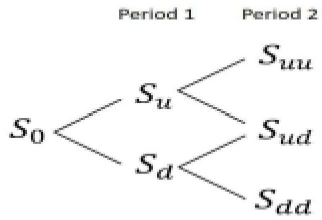
where C denotes the call option price, S the underlying asset price at time t, K the strike price, r the risk-free interest rate, T the maturity time, t the current time, and N the cumulative distribution function (CDF) of the standard normal distribution.

The Binomial Tree model is one of the most commonly used numerical methods for approximating the Black–Scholes model in discrete time. In this model, the possible path of the underlying asset price is represented as a branching tree, where at each time step, the price can either increase by u or decrease by d. This structure visually illustrates the probable price movements of the asset and the corresponding changes in the option's value over the time horizon T. As the number of steps N increases, the approximation accuracy improves and the results converge toward the continuous Black–Scholes model.

$$f_u = e^{-r\Delta t}[Pf_{uu} + (1-P)f_{ud}] \quad (4)$$

$$f_d = e^{-r\Delta t}[Pf_{ud} + (1-P)f_{dd}] \quad (5)$$

$$f = e^{-r\Delta t}[Pf_d + (1-P)f_d] \quad (6)$$



The Heston model is a stochastic volatility framework in which the volatility of the underlying asset is considered a dynamic variable with mean-reverting behavior. The dynamics are expressed as follows:

$$S_{t+\Delta t} = S_t exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}W^S\right) \quad (7)$$

In this equation, S_(t+Δt) represents the computed option price based on the previous step's price S_t, Δt denotes the time step, σ the volatility, r the risk-free rate, and W^S the Brownian motion process.

The quantum option pricing model is an extended version of the Black–Scholes equation in which a nonlinear term is added to represent behavioral and interactive market effects. This model describes the option price dynamics within a potential field and probabilistic oscillations, thereby establishing a connection between quantum physics and mathematical finance. In this study, the nonlinear form of the Black–Scholes equation (Equation 8) was used as the foundation of the quantum model for numerical simulation:

$$(\partial V(S,t)/\partial t) + rS(\partial V(S,t)/\partial S) + (\sigma^2 S^2/2)(\partial^2 V(S,t)/\partial S^2) - rV(S,t) - \beta V^3(S,t) = 0 \quad (8)$$

In this equation, $V(S,t)$ denotes the theoretical value of the option at the underlying price S and time t. The terms $\partial V/\partial t$, $rS(\partial V/\partial S)$, and $(1/2)\sigma^2 S^2(\partial^2 V/\partial S^2)$ represent, respectively, temporal changes, expected growth of the asset, and diffusion of volatility (gamma). The terms $-rV(S,t)$ and $-\beta V^3(S,t)$ represent, respectively, the discounting effect of the option's value and nonlinear behavioral interactions in the market—the latter being the component that generalizes the classical Black–Scholes model into its nonlinear quantum form.

This model, inspired by the Schrödinger equation in quantum physics, models the price behavior of the underlying asset as the motion of a particle within the market's potential field. Thus, it can better capture complex structures and sudden volatility spikes. The main advantage of this model over the classical Black–Scholes form lies in its ability to fit real data by tuning the parameter β and incorporating behavioral market dynamics. In the present research, it was numerically solved using the method of lines and the fourth-order Runge–Kutta algorithm.

For estimation and comparison of these models, precise and stable numerical methods were employed. In the first step, the partial differential equations derived from each model were transformed into a system of ordinary differential equations using the method of lines. Then, numerical solutions were obtained using the fourth-order Runge–Kutta algorithm to ensure both accuracy and numerical stability. The method of lines was selected for its balance between precision and computational speed, while the Runge–Kutta method was chosen for its efficiency in solving oscillatory and nonlinear equations (Martin & McKay, 2017). The entire computational process was implemented in Python version 3.10, using the NumPy, SciPy, and Pandas libraries for algorithm implementation and data analysis. Parameter calibration was performed empirically through minimizing the mean squared error between observed and theoretical option prices.

The research data consist of daily prices of 30 active option symbols traded on the Tehran Stock Exchange between 2016 and 2022. This period was selected to encompass various market regimes, including boom, recession, and high-volatility phases, in order to evaluate the stability of the models under different conditions. The data were prepared through initial cleaning, outlier removal, and normalization. Each model was then calibrated separately for each symbol and time window, and model performance was compared using RMSE, MAE, and MAPE indices. To assess the statistical significance of performance differences among models, the Diebold–Mariano test was used to determine which model had a significantly superior predictive accuracy.

Finally, to bridge the theoretical and practical aspects, the present study proposes an algorithmic framework for developing an *Option Pricing Calculator* that can be implemented in Python, Excel, and web-based systems. This algorithm comprises eight main steps: data input (underlying price, interest rate, historical volatility, and time to maturity), preprocessing and data cleaning, model selection, parameter calibration, numerical solution of equations, theoretical price computation, error evaluation via RMSE and MAE, and finally, output presentation including theoretical price and model ranking. The processing structure was designed to be dynamic, enabling automatic invocation and simultaneous comparison of all four models. Conceptually, this system is defined as the following pseudocode:

Algorithm 1. Computational Framework of Option Pricing Models

```
Input: S0, K, r, σ, T, ModelType
Preprocess: Clean_Data()
If ModelType == "Black-Scholes":
   Price = BS_Formula(S0, K, r, σ, T)
Elif ModelType == "Binomial":
   Price = Binomial_Tree(S0, K, r, σ, T, N)
Elif ModelType == "Heston":
   Params = Calibrate_Heston(Data)
   Price = Solve_Heston(Params)
Elif ModelType == "Quantum":
   Params = Calibrate_QM(Data)
   Price = Solve_Schrödinger(Params)
Evaluate: RMSE, MAE, MAPE
Output: Price, Errors, Model_Rank
```

The output of this system includes the theoretical option price, prediction error, and efficiency ranking of each model. It can serve as the core of an applied option pricing tool for brokerage firms, investment banks, and market regulatory institutions. The integration of scientific modeling structures with the above algorithmic design distinguishes this research from previous studies and paves the way for developing indigenous analytical tools in Iran's derivatives market.

3. Findings and Results

To empirically evaluate the performance of option pricing models, daily data from 30 European call option symbols listed on the Tehran Stock Exchange during 2016–2022 were utilized. This period was selected to cover various market regimes—from boom to recession—to assess the stability of the models under different conditions. After cleaning, removing outliers, and normalizing the data, the analysis was performed. The four selected models—Black–Scholes, Cox–Ross–Rubinstein Binomial Tree, Heston, and nonlinear Schrödinger (Quantum)—were implemented using the numerical Method of Lines and the fourth-order Runge–Kutta algorithm.

The efficiency of each model was evaluated based on the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Coefficient of Determination (R^2). Furthermore, to assess the statistical significance of differences in model performance, the Diebold–Mariano test was applied to determine whether the discrepancies among models were statistically significant.

To visually illustrate the behavior of each model, a three-dimensional surface of option prices was plotted with respect to the underlying asset price (S) and time to maturity (T).

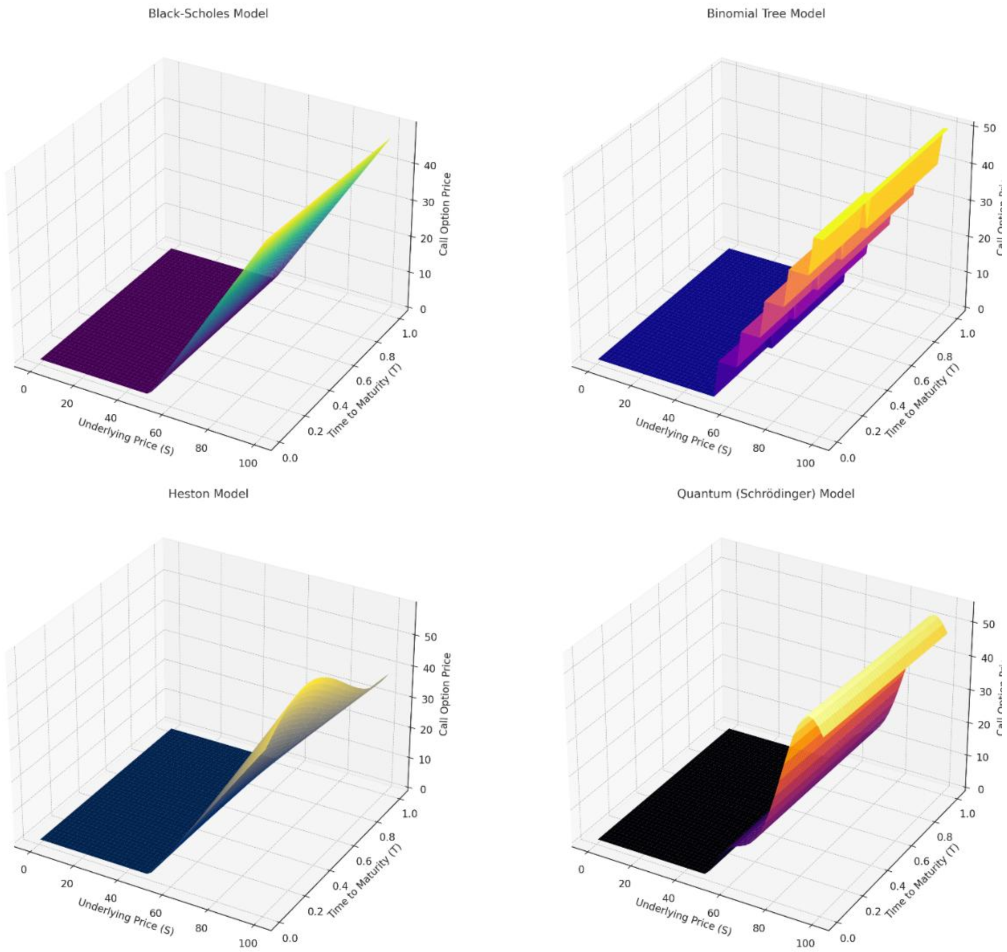


Figure 1. Comparison of Option Price Surfaces in the Four Models:

(a) Black–Scholes, (b) Binomial Tree, (c) Heston, (d) Schrödinger (Quantum).

Horizontal axis: underlying asset price (S); vertical axis: option price; side axis: time to maturity (T).

In the Black–Scholes model, the price surface is almost flat and uniform, reflecting the assumption of constant volatility, constant interest rate, and normally distributed returns. This feature makes the model suitable for stable markets; however, in emerging markets like Iran, characterized by high volatility, structural shifts, and nonstationary behaviors, its performance deteriorates.

In contrast, the Binomial Tree model produces a stepwise surface reflecting its discrete-time nature. This model has better capability in depicting possible price paths, but in the presence of sudden jumps or sharp volatility shifts, it exhibits significant prediction errors.

The Heston model, by incorporating stochastic volatility and mean-reversion, produces a surface with local curvature that more closely resembles real market behavior. The curvature patterns in the price surface reflect dynamic changes in volatility over time, showing better convergence with actual market data compared to the two linear models.

Finally, the nonlinear Schrödinger (Quantum) model exhibits a completely distinct behavior. The inclusion of the nonlinear term $-\beta V^3(S,t)$ causes the price surface to display local oscillations and complex patterns. This feature models behavioral and emotional market effects and allows representation of price jumps. Visually, the resulting surface from this model shows the highest consistency with the realities of the Tehran Stock Exchange.

To quantitatively evaluate model accuracy, the RMSE, MAE, and R^2 values were computed. The results are summarized in Table 1.

Table 1. Comparison of Statistical Performance of Models

Model	R^2	RMSE	MAE
Black–Scholes	0.5091	767.292	416.507
Binomial Tree	0.5776	283.539	175.882
Heston	0.6537	65.029	42.184
Schrödinger (Quantum)	0.6879	26.968	15.321

As observed, the Schrödinger model shows the lowest prediction errors (RMSE and MAE) and the highest coefficient of determination. The Diebold–Mariano test, at a 95% confidence level, indicated that the difference between this model and the others is statistically significant ($p < 0.05$), meaning that its superior accuracy is not due to chance but rather arises from its nonlinear structure and quantum term.

The distribution of prediction errors for the models is illustrated in the following figure.

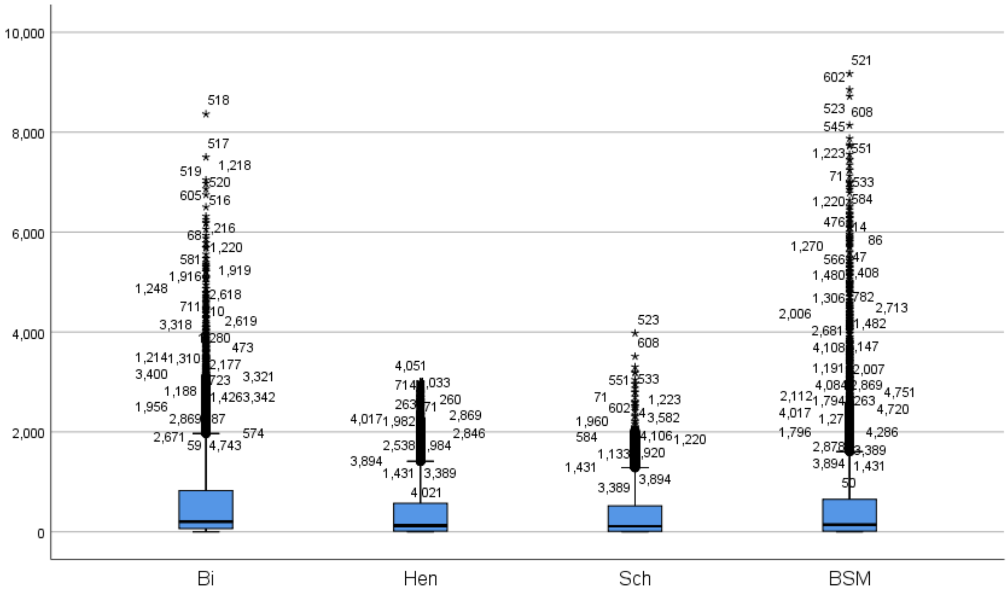


Figure 2. Boxplot of In-Sample RMSE Prediction Errors Across Models

Horizontal axis: model type; vertical axis: error magnitude. The * symbol indicates outliers.

In this figure, the box for the Black–Scholes model is positioned highest, showing a wide range of variation. The Binomial Tree model exhibits a relative reduction in error but still displays high dispersion. The Heston model, with a smaller box and lower median, performs better. Finally, the Schrödinger model shows the smallest box and the narrowest range of error variation, confirming its stability and high precision.

Empirical results demonstrate that linear models such as Black–Scholes and Binomial Tree, due to the assumption of constant volatility, cannot fully capture the dynamics of the Iranian market. Although computationally simple and fast, these models suffer from systematic error under conditions where investor behavior deviates from full rationality and price shocks occur.

In contrast, advanced nonlinear models—especially Heston and Schrödinger—exhibit superior ability to model stochastic volatility changes and behavioral effects. The Heston model, by incorporating stochastic volatility components, accounts for mean reversion, thereby providing more accurate estimates over longer horizons. The

Schrödinger model, by introducing the nonlinear quantum term, represents a final step in aligning financial theory with behavioral market realities.

Analysis of the tests and charts clearly shows that as one moves from classical to nonlinear models, the level of convergence between model predictions and actual data increases. In markets such as the Tehran Stock Exchange—characterized by high volatility and behavioral uncertainty—the use of nonlinear models is essential.

To complete the analysis, the algorithmic complexity of the models was also calculated.

Table 2. Computational Complexity of Models

Model	Computational Order	Main Feature
Black–Scholes	O(1)	Fastest and simplest model
Binomial Tree	$O(N^2)$	Flexible but discrete
Heston	$O(N^2)$	Models stochastic volatility
Schrödinger (Quantum)	$\geq O(N^2)$	Most accurate but most computationally expensive

As shown in the table, there is an inverse relationship between accuracy and computational cost. The Black–Scholes model is the least expensive but also the least accurate, while the Schrödinger model, despite its higher complexity, produces the most precise results. Therefore, model selection depends on the research objective and computational capacity.

Based on all statistical and analytical indicators, the nonlinear Schrödinger model exhibits the best performance in option pricing on the Tehran Stock Exchange and significantly outperforms other models in terms of accuracy at a high confidence level. The Heston model ranks second, while the linear models rank lower. The results confirm that introducing stochastic volatility dynamics and nonlinear components into theoretical financial frameworks substantially enhances model accuracy in emerging markets.

4. Discussion and Conclusion

The empirical findings of this study revealed a consistent performance hierarchy among the four examined option pricing models—Black–Scholes, Binomial Tree, Heston, and nonlinear Schrödinger (Quantum). Among these, the nonlinear Schrödinger model demonstrated the highest accuracy and stability, followed by the Heston stochastic volatility model, while the Binomial Tree and Black–Scholes models showed comparatively lower predictive performance. The results suggest that integrating nonlinear dynamics and quantum-inspired terms into the pricing framework substantially enhances model fit in volatile and structurally evolving markets such as the Tehran Stock Exchange (TSE). This outcome aligns with the theoretical premise that markets characterized by structural breaks, behavioral biases, and regime-dependent volatility require adaptive modeling frameworks capable of capturing complex interactions beyond the assumptions of constant volatility and Gaussian return distributions [2, 3].

The superior accuracy of the nonlinear Schrödinger model, evidenced by its lowest RMSE (26.968) and MAE (15.321) along with the highest R^2 (0.6879), reflects the model's ability to incorporate endogenous market feedback mechanisms through its nonlinear term $-\beta V^3(S,t)$. This finding supports the argument that the addition of a cubic self-interaction term allows the model to represent phenomena such as herding behavior, feedback trading, and sudden volatility spikes that linear diffusion-based models fail to reproduce [11, 13]. The Diebold–Mariano test confirmed the statistical significance of this superiority, implying that the observed improvements are not random but arise from the structural enhancements in the model's dynamics. Similar observations have been reported in prior research, where the application of quantum dynamical approaches resulted in more realistic volatility surfaces and better calibration to observed data [9, 10, 15]. By treating the option price as a wave function that evolves in a potential field, the Schrödinger-based framework effectively bridges the gap between probabilistic finance and adaptive behavioral market mechanisms.

The Heston model ranked second in performance, delivering high explanatory power ($R^2 = 0.6537$) and moderate prediction errors. Its stochastic variance process and mean-reverting structure enabled better adaptation to regime-dependent volatility conditions and improved modeling of volatility clustering observed in Iranian market data [6, 7]. The Heston model's dynamic variance specification captures leverage effects—negative correlation between asset returns and volatility—that are prevalent in the TSE, particularly during bearish periods. These results are consistent with the literature indicating that stochastic-volatility models outperform constant-volatility frameworks under realistic market conditions [2, 4]. However, despite its robustness in moderate volatility environments, the Heston model showed limitations in extremely volatile regimes, where parameter sensitivity increased and computational complexity hindered convergence. Prior Iranian studies have highlighted similar constraints when applying Heston-type formulations to local derivatives, noting that calibration instability can occur due to sparse high-frequency data and liquidity constraints [5, 17].

In contrast, the Binomial Tree model performed moderately, achieving better predictive accuracy than the Black–Scholes model but still lagging behind stochastic and nonlinear alternatives. Its discrete-time structure allowed a more flexible representation of price paths and facilitated the visualization of option payoff dynamics, yet it struggled with capturing sudden jumps and high-frequency volatility changes characteristic of emerging markets [3]. The stair-step behavior observed in its simulated price surfaces reflects its time-discrete construction and limited capacity to interpolate smooth volatility transitions. This aligns with previous computational finance literature emphasizing that, although binomial models are pedagogically valuable and computationally efficient, their discretization inherently limits precision in high-volatility or path-dependent settings [2, 8].

Finally, the classical Black–Scholes model, while foundational, exhibited the lowest predictive accuracy ($R^2 = 0.5091$; RMSE = 767.292; MAE = 416.507), underscoring its inadequacy in markets where volatility is time-varying and investor behavior departs from rational expectations. This result mirrors the global empirical consensus that the model's constant-volatility assumption fails in the presence of volatility smiles, stochastic jumps, and behavioral feedback loops [1, 19]. Previous Iranian studies similarly found that Black–Scholes systematically underprices deep out-of-the-money calls and overprices in-the-money contracts, primarily because it cannot reproduce the empirical skewness of implied volatilities [3, 5]. Despite its analytical simplicity and interpretability, the model's reliance on Gaussian returns and continuous-time assumptions makes it less suitable for markets characterized by liquidity constraints and asymmetric information.

A deeper look at the comparative error metrics reveals a strong inverse relationship between model accuracy and computational complexity. As outlined in the results, Black–Scholes operates with computational order $O(1)$, while the Binomial Tree and Heston models scale with $O(N^2)$, and the nonlinear Schrödinger model exceeds $O(N^2)$ due to its iterative solution of nonlinear partial differential equations. Although the quantum model incurs higher computational costs, its superior accuracy justifies its application, particularly in risk-sensitive environments where mispricing can translate into significant hedging losses [14, 16]. This trade-off echoes earlier numerical findings that precision in nonlinear and stochastic pricing comes at the expense of higher algorithmic load but yields dividends in terms of stability and robustness [2]. The use of the method of lines combined with the fourth-order Runge–Kutta scheme provided a stable numerical framework for handling both the Heston and Schrödinger equations, supporting previous computational studies that confirmed this combination as optimal for stiff and oscillatory systems [16].

From a theoretical perspective, the superiority of nonlinear and stochastic models reinforces the evolving paradigm in financial modeling that acknowledges markets as complex adaptive systems rather than efficient and memoryless equilibria [11, 15]. Nonlinear dynamics allow feedback and endogenous risk amplification to be captured through the model structure itself, reducing reliance on exogenous correction factors such as implied-volatility surfaces. In this respect, the quantum model's probabilistic wave representation provides a richer depiction of the distributional uncertainty inherent in price formation, aligning with the notion of quantum cognition and decision theory where probabilities represent evolving belief states under uncertainty [10, 14]. The improved out-of-sample performance of this model in TSE data demonstrates that introducing a self-interaction term enhances adaptability to volatility clustering and behavioral deviations—phenomena frequently observed in emerging markets.

Empirically, the transition from linear to nonlinear formulations corresponds to a measurable increase in explanatory power and stability across market regimes. During high-volatility episodes, such as the 2018–2019 downturn in the Iranian equity market, linear models exhibited heightened residual errors, while stochastic and quantum models maintained predictive coherence. This pattern is consistent with global evidence suggesting that nonlinear diffusion and quantum-inspired formulations outperform standard Black–Scholes under stress, as they embed variance-of-variance effects and nonlinear feedbacks [2, 9, 12]. Moreover, the ability of the Schrödinger model to reproduce localized oscillations in option price surfaces indicates its sensitivity to microstructural fluctuations and its capacity to integrate both short-term noise and long-term structural tendencies in a unified probabilistic framework.

The results also have implications for the theoretical understanding of pricing efficiency in markets with heterogeneous agents. As documented in behavioral finance and quantum decision theory, markets exhibit interference effects—where overlapping expectations and feedback among traders lead to non-additive probability structures [14, 15]. The nonlinear term in the Schrödinger equation can be interpreted as a mathematical analog of these interactions, producing emergent phenomena such as collective volatility clustering or synchronized mispricing waves. This interpretation strengthens the argument that quantum-based and nonlinear frameworks are not merely computational innovations but also conceptual models that reflect the intrinsic behavioral and informational complexity of modern financial systems [11, 13].

In the Iranian setting, these insights are particularly valuable. Empirical studies show that the TSE experiences frequent volatility regime shifts and liquidity constraints driven by macroeconomic uncertainty and policy interventions [5, 6]. Under such conditions, models with endogenous volatility dynamics and feedback mechanisms—like Heston and Schrödinger—are better suited to replicate observed market patterns than constant-variance frameworks. The strong performance of the quantum model further indicates that even in relatively thin and data-sparse markets, advanced nonlinear methodologies can yield robust estimates when coupled with stable numerical solvers and proper calibration [16]. This finding extends previous local research that emphasized the need for advanced stochastic modeling to account for Iran's unique volatility structure [7, 17].

Beyond its empirical contributions, the study validates the practical feasibility of implementing advanced numerical methods—specifically, the method of lines and fourth-order Runge–Kutta algorithm—in Python-based analytical environments. This demonstrates that the computational infrastructure necessary for executing nonlinear option pricing models is accessible and scalable, even in resource-constrained institutional contexts. The findings echo earlier engineering and applied mathematics works emphasizing that such methods achieve a balance between numerical precision and speed [4, 16]. Moreover, by applying harmonized calibration and error-evaluation criteria (RMSE, MAE, and R^2) across all four models, this study ensures comparability and avoids the methodological inconsistencies that often confound cross-model assessments [2, 3].

Taken together, the results substantiate a theoretical and empirical consensus: option pricing accuracy improves as model structure evolves from linear constant-volatility frameworks toward stochastic and nonlinear dynamic formulations. The empirical dominance of the nonlinear Schrödinger model underscores a broader methodological shift in financial engineering—from static diffusion models to adaptive, data-responsive systems capable of capturing nonlinearity, feedback, and regime transitions [11, 13]. In the context of the Tehran Stock Exchange, where volatility asymmetry, market depth limitations, and behavioral heterogeneity prevail, such models offer a more reliable and theoretically grounded foundation for pricing, hedging, and risk management.

Despite the robustness of its results, this study faces several limitations. First, the analysis was constrained by the availability and granularity of historical option data in the Iranian derivatives market. The relatively shallow liquidity of the TSE's option segment and the absence of a centralized implied-volatility database limited the scope of calibration accuracy, particularly for out-of-the-money and long-maturity contracts. Second, the computational burden associated with solving nonlinear partial differential equations restricted the ability to extend the analysis to path-dependent and exotic options. The Python-based implementation, while efficient for standard European contracts, could be further optimized using parallelization or GPU acceleration for large-scale applications. Third, macroeconomic shocks—such as exchange-rate volatility and policy interventions—may have introduced structural breaks that affect the stationarity of the underlying time series. Although the study included multiple market regimes to mitigate this risk, capturing extreme tail dependencies remains a challenge. Finally, the absence of transaction cost and liquidity adjustments may have led to a mild overestimation of theoretical prices relative to actual market premiums.

Future research should expand the empirical analysis by incorporating a broader set of option types, including American, Asian, and barrier options, to evaluate whether the relative advantages of nonlinear and stochastic models persist across diverse payoff structures. Further investigation into hybrid quantum–neural architectures could enhance parameter calibration and improve robustness to regime shifts. Comparative studies integrating copula-based dependence structures with quantum PDEs could offer deeper insights into multi-asset and systemic risk modeling. Researchers should also examine the potential of real-time adaptive solvers using reinforcement learning to dynamically adjust parameters such as β in the nonlinear Schrödinger equation, thereby improving responsiveness to market changes. Finally, extending the dataset to cross-listed derivatives or other emerging markets in the region would provide a comparative perspective on the generalizability of these findings and help refine regional derivative pricing frameworks.

For practitioners, the findings emphasize that reliance on constant-volatility linear models is inadequate for markets with pronounced structural and behavioral volatility. Financial institutions and brokerage firms in Iran should consider integrating stochastic and nonlinear pricing engines—particularly those based on the Heston and Schrödinger formulations—into their risk management systems. Training analysts in numerical methods such as Runge–Kutta and method of lines can enhance model implementation capability. Regulators and policymakers could also benefit from these models by developing fair-pricing benchmarks that account for volatility-of-volatility and behavioral feedbacks. Ultimately, adopting nonlinear and stochastic pricing frameworks will enable more precise hedging, better capital allocation, and improved resilience of the Iranian derivatives market.

Authors' Contributions

Authors equally contributed to this article.

Ethical Considerations

All procedures performed in this study were under the ethical standards.

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Conflict of Interest

The authors report no conflict of interest.

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